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ABSTRACT

This trigonometry curriculum guide, produced under the direction of the State of Louisiana's Department of Public Education, is a segment of the educational program established in response to accountability, assessment, and competency-based education laws. This guide is designed to represent the best thinking of a selected statewide committee established to determine the scope of mathematics content for a course in trigonometry at the secondary school level. The guide contains: (1) the membership rosters of the committees involved in developing the material: (2) a review of the curriculum development process: (3) eleven required goals that students completing the trigonometry course should be able to reach: (4) a complete curriculum outline with performance objectives: and (5) a detailed set of activities grouped with specific objectives and content areas of the trigonometry program. (MP)

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Trigonometry

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J. KELLY NIX
State Superintendent

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FOREWORD

Curriculum guides have been developed for each mathematics course at the secondary level and for grades K-8 at the elementary level. These guides represent the best thinking of a selected statewide committee established to determine the scope of mathematics content which should be taught at each level.

The mathematics curriculum guides are another segment of the total educational program established by this administration and mandated by the Legislature in both the accountability and assessment and the competency-based education laws. This educational program requires that specific skills and concepts be established for each grade level and for each subject area. The mathematics curriculum guides with course outlines, performance objectives and coordinated activities effect this phase of the program.

It is hoped that the guides will make a major contribution to the improvement of mathematics instruction in the schools of Louisiana. This is another step toward achieving the goals of this administration.

FOR OUR CHILDREN

J. KELLY NIX

ACKNOWLEDGMENTS

The statewide mathematics committee is to be commended for its work in the development of the Mathematics Curriculum Guides K-12. The committee worked under the chairmanship of Dr. Jean Reddy, Section Chief of the Mathematics Section in the Bureau of Secondary Education.

The Bureaus of Elementary Education and Secondary Education were responsible for writing the activities component of the Mathematics Curriculum Guides. The elementary supervisors in the Bureau of Elementary Education with Mrs. Bonnie Ross serving as chairman of the committee, developed the activities for the K-8 guide. The activities for the secondary guides were drafted by a committee under the leadership of Dr. Jean Reddy. These people are to be commended for their colossal accomplishments in this formidable project.

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INTRODUCTION

Act 750 of the 1979 Louisiana Legislature established the Louisiana Competency-Based Education Program. One of the most important provisions of Act 750 is the mandated "development and establishment of statewide curriculum standards for required subjects for the public elementary and secondary schools of this state...." The "statewide curriculum standards for required subjects" is defined as "the required subjects to be taught, curriculum guides which contain mimimum skills and competencies, suggested activities, suggested materials of instruction, and minimum required time allotments for instruction in all subjects." Act 750 further provides that the "effective implementation date of the statewide curriculum standards for required subjects shall be the 1981-82 school year. Development of such curriculum shall begin by the 1979-80 school year."

During the 1978-79 school year, curriculum guides were developed by advisory and writing committees representing all levels of professional education and all geographic areas across the State of Louisiana for the following mathematics courses:

> Algebra I, Algebra II, Geometry, Advanced Mathematics, and Trigonometry.

The major thrust of the curriculum development process in each of the guides has been the establishment of minimum standards for student achievement. Learning expectancies for mastery have been determined for each course and/or grade level. In addition, content outlines, suggested activities, procedures, and bibliographies have been developed as aids in support of the learning expectancies. The curriculum guides also contain activities designed to stimulate learning for those students capable of progressing beyond the minimums.

During the 1979-80 school year, the curriculum guides were piloted by teachers in school systems representing the different geographic areas of the state as well as urban, suburban, inner-city, and rural schools. The standard populations involved in the piloting reflected also the ethnic composition of Louisiana's student population. Participants involved in the piloting studies utilized the curriculum guides to determine the effectiveness of the materials that were developed. Based upon the participants' recommendations at the close of the 1979-80 pilot study, revisions were made in the curriculum guides to ensure that they are usable, appropriate, accurate, comprehensive, relevant, and clear.

The curriculum guides are now ready for full program implementation. This stage must be understood in its operational context. The curriculum developers and the participants in the pilot studies do not stand alone in promoting learning expectancies that will improve education for the students in the State of Louisiana. Ultimately, local system supervisors, principals, and classroom teachers will have the responsibility for attaining this goal.

As curriculum guides are implemented, the following guidelines should prove helpful:

> ...curriculum standards should be considered as the foundation for the year's instructional program. Where other programs are already in operation, these curricular materials must be checked with the foundation curricula to ensure that appropriate course and/or grade level standards are included and maintained.

...curricular activities contained in the guides provide a number of suggestions for helping students to achieve the established standards. Activities to meet the needs of "average," "below average," and "above average" students have been included. These activities should prove helpful as the teacher plans and organizes instruction. Additional activities, however, may supplement or be used in lieu of those listed in the guide as long as these activities are designed to achieve similar specific objectives.

...curricular suggestions for meeting the needs of the special child have been prepared by the Division of Special Education. These suggestions are designed to provide help for teachers who work with special children in the regular classroom.

The continued effort of mathematics teachers to provide quality instruction will enhance our statewide goal to ensure that every student in the public elementary and secondary schools of the State of Louisiana has an opportunity to attain and to maintain skills that are considered essential to functioning effectively in society.

J. KELLY NIX
State Superintendent of Education

GOALS

Upon completing a high school course in trigonometry, a student will be able to:

1. Acquire a knowledge of an angle and its measure as applicable to trigonometry.
2. Know basic trigonometry definitions and formulas and be able to apply them.
3. Find the trigonometric functions of any given angle.
4. Graph the six trigonometric functions and their variations.
5. Solve problems involving inverse trigonometric relations.
6. Use identities in solving trigonometric equations.
7. Understand the system of polar coordinates.
8. Know the properties of complex numbers.
9. Solve triangles.
10. Understand the relationship between trigonometric functions and circular functions.
11. Acquire a knowledge and understanding of circular functions.

CURRICULUM OUTLINE AND PERFORMANCE OBJECTIVES

NOTE: All items are mandatory unless preceded by an asterisk. All items with an asterisk should be taught if time permits (See Pacing Chart).

CURRICULUM OUTLINE	PERFORMANCE OBJECTIVE
I. Circular Functions	To display an understanding of circular functions, the student will be able to:
A. Unit circle	A. Define the unit circle and demonstrate symmetry with respect to the two axes and the origin.
B. Wrapping function	B. Define and illustrate the wrapping function.
C. Sine and cosine functions	C. Define sine and cosine functions as coordinates of a point on the unit circle and state the domain, range and fundamental period.
D. Graphs of functions of the form $y = \sin x$ and $y = \cos x$	D. Graph two or more periods of functions of the form $y = \sin x$ and $y = \cos x$.
E. Graphs of functions of the form $y = a \sin bx$ and $y = a \cos bx$	E. Graph functions of the form $y = a \sin bx$ and $y = a \cos bx$ by finding the period and/or amplitude of each.
F. Graphs of functions of the form $y = a \sin (bx - c) + d$ and $y = a \cos (bx - c) + d$	F. Graph functions of the form $y = a \sin (bx - c) + d$ and $y = a \cos (bx - c) + d$ by finding the period and/or amplitude of each.
G. Other circular functions	G.
	1. Define tangent, cotangent, secant and cosecant in terms of the sine and cosine;
	2. Graph and state the domain and range of each.
H. Identities	H. State the eight basic identities and use them to verify other identities.

CURRICULUM OUTLINE	PERFORMANCE OBJECTIVES
II. Angles And Their Measures	To demonstrate a knowledge of basic definitions, the student will be able to:
A. Definition of an angle	A. Define an angle and identify the initial and terminal sides of an angle.
B. Standard positions of an angle	B. Sketch angles in standard position.

CURRICULUM OUTLINE	PERFORMANCE OBJECTIVES
III. Radian Measure	To demonstrate an understanding of degree measure and radian measure, the student will be able to:
A. Conversions	A. Convert radian measure to degree measure and degree measure to radian measure.
B. Length of an arc of a circle	B. Use radian measure to find the length of an arc of a circle;
C. Area of sector	C. Use radian measure to find the area of a sector of a circle;
D. Velocity	D.
	1. Compute linear speed of a point moving on a circle;
	2. Find angular velocity of a point moving on a circle.

CURRICULUM OUTLINE	PERFORMANCE OBJECTIVES
IV. Trigonometric Functions	To exhibit a knowledge of a trigonometric functions, the student will be able to:
A. Functions of angles	A. Write the six trigonometric functions of any angle.
B. Functions of special angles	B. Write the trigonometric functions of angles whose measures are 30, 45, and 60 degrees.
C. Cofunctions	C. Write functions of acute angles as functions of the complement of an angle.
D. Reciprocal relations	D. Write functions of angles as reciprocal functions of the same angle.
E. Quotient and Pythagorean relations	E. Write trigonometric expressions as equivalent expressions by using the quotient and Pythagorean relations.
F. Tables and linear interpolation	F. 1. Find functions of angles by reading from a table; 2. Find functions of angles by linear interpolation.

CURRICULUM OUTLINE	PERFORMANCE OBJECTIVES
V. Trigonometric Functions of Any Angle	To demonstrate a knowledge of special and quadrantal angles, the student will be able to:
A. Reference angles	A. Express the functions of any angle as a function of a reference angle.
B. Quadrantal angles	B. Find the functions of angles whose measures are multiples of 0, 90, 180, and 270 degrees.
C. Formulas	C.
1. Sum and difference formulas	1. Apply the formulas for the functions of the sum or difference of two angles;
2. Half-angle and double-angle formulas	2. Apply the double-angle and half-angle formulas.
D. Identities	D. Verify that expressions are identities by using trigonometric formulas.

CURRICULUM OUTLINE	PERFORMANCE OBJECTIVES
VI. Inverse Trigonometric Relations	To display an understanding of the inverse trigonometric relations, the student will be able to:
A. General value	A. Find general values of inverse relations.
B. Graph of inverse	B. Graph inverse trigonometric functions.
C. Principal values of inverse functions	C. Find principal values of inverse trigonometric functions.

CURRICULUM OUTLINE	PERFORMANCE OBJECTIVES
VII. Trigonometric Equations	To demonstrate an understanding of trigonometric identities and equations, the student will be able to:
A. Principal value	A. Find principal values of trigonometric equations.
B. General values	B. Find general values of trigonometric equations.
C. Restricted solutions	C. Find restricted solutions of trigonometric equations.

CURRICULUM OUTLINE	PERFORMANCE OBJECTIVES
VIII. Applications of Trigonometry	To exhibit an understanding of the use of trigonometric ratios in finding missing parts of triangles, the student will be able to:
A. Right triangle	A. 1. Solve right triangles given two sides and an angle; 2. Solve right triangles given two angles and a side.
B. Law of sines and law of cosines	B. Solve triangles by applying the law of sines and the law of cosines.
C. Use of calculators	C. Perform calculations involving trigonometric functions using the calculator.
D. Area of triangles	D. Find the area of a triangle given the lengths of three sides or the lengths of two sides and the measure of the included angle of the triangle.

CURRICULUM OUTLINE		PERFORMANCE OUTLINE
IX.	Polar Coordinates	To display an understanding of the polar coordinate system, the student will be able to:
	A. Conversion	A. Convert from rectangular to polar coordinates and vice-versa.
	B. Graphing	B. Sketch graphs of relations in polar form.

CURRICULUM OUTLINE	PERFORMANCE OBJECTIVES
X. Complex Numbers	To exhibit a knowledge of complex numbers, the student will be able to:
A. Definition and properties	A.
	1. Define a complex number;
	2. Perform fundamental operations on complex numbers.
B. Rectangular and polar forms	B. Write complex numbers in rectangular and polar form.
C. Multiplication and division in polar form	C. Multiply and divide complex numbers in polar form.
D. DeMoivre's theorem	D. Use DeMoivre's Theorem to raise complex numbers to integral powers.
E. Roots of complex numbers	E. Find all roots of a complex number.

ACTIVITIES

I. A. CONTENT: Circular Functions; Unit Circle

OBJECTIVE: The student will be able to:

(a) Define a unit circle;

(b) Demonstrate symmetry with respect to the axes and the origin.

ACTIVITIES:

(a) Define a unit circle.

(b) Given a point (a,b) on a unit circle, determine the coordinates of a point on the unit circle which demonstrates symmetry with respect to:

(1) The horizontal axis.

(2) The vertical axis.

(3) The origin.

I. B. CONTENT: Circular Functions; Wrapping Functions

OBJECTIVE: The student will be able to:

(a) Define the wrapping functions;

(b) Specify the domain and range of the wrapping function.

ACTIVITIES:

(a) Define a wrapping function.

(b) Given w(x) to be the wrapping function find the following and state the domain and range of each.

(1) $w(\frac{\pi}{2})$

(2) $w(\pi)$

(3) $w(3\pi)$

(4) $w(-\frac{\pi}{2})$

(5) $w(\frac{5\pi}{2})$

(c) In which quadrant does w(x) lie, where x equals:

(1) -3

(2) + 3

(3) $\frac{7\pi}{5}$

(4) $\frac{3}{2}$

(5) $\frac{10\pi}{3}$

(6) -25

I. C. CONTENT: Circular Functions; Sine and Cosine Functions

OBJECTIVE: The student will be able to:

(a) Define the sine functions as a set of all ordered pairs (x,v) where v = sin x, and the cosine functions as a set of all ordered pairs (x,u) where u - cos x;

(b) State the domain and range;

(c) State the fundamental period;

(d) Find the values of the sine and cosine of integral multiples of $\frac{\pi}{6}$, and $\frac{\pi}{4}$ and $\frac{\pi}{3}$.

ACTIVITIES:

(a) Sin x = $\frac{4}{5}$ and x lies in the second quadrant, find cos x.

(b) State the domain and range of the sine and cosine function.

(c) Name the numbers x, such that sin x = 1.

(d) Find the values of the following:

(1) cos $\frac{5\pi}{3}$

(2) sin $\frac{5\pi}{4}$

(3) sin $\frac{5\pi}{6}$

(4) cos $\frac{17\pi}{3}$

(5) $\sin \frac{-23\pi}{4}$

I. D. CONTENT: Circular Functions; Graphs of Functions of the form $y = \sin x$ and $y = \cos x$

OBJECTIVE: The student will be able to sketch two or more periods of $y = \sin x$ and $y = \cos x$.

ACTIVITIES: Sketch the graphs of each of the following over the indicated interval.

(a) $y = \sin x$ from -2π to 2π.

(b) $y = \cos x + \sin x$ from 0 to π.

(c) $y = \cos x$ from $\frac{-\pi}{2}$ to $\frac{5\pi}{2}$.

(d) $y = |\cos x|$ from -2π to 2π.

I. E. CONTENT: Circular Functions; Graphs of Functions of the form $y = a \sin bx$ and $y = a \cos bx$

OBJECTIVE: The student will be able to graph functions of the form $y = a \sin bx$ and $y = a \cos bx$ by finding the amplitude and/or period of each.

ACTIVITIES:

(a) Find the amplitude and period of each of the following:

(1) $y = 2 \sin 3x$.

(2) $y = 4 \cos x$.

(3) $y = -3 \cos \frac{1}{3}x$.

(4) $y = \frac{1}{4} \cos \frac{1}{4}x$.

(b) Sketch two periods of the graphs of:

(1) $y = 2 \sin 3x$.

(2) $y = 3 \cos \frac{1}{2}x$.

(3) $y = 3 \sin \pi x$.

(4) $y = 3 \sin \frac{1}{3}x$.

I. F.

CONTENT: Circular Functions; Graphs of Functions of the form $y = a \sin (bx - c) + d$ and $y = a \cos (bx - c) + d$

OBJECTIVE: The student will be able to graph functions of the form $y = a \sin (bx - c) + d$ and $y = a \cos (bx - c) + d$ by finding the amplitude and/or period of each.

ACTIVITIES: Sketch two periods of the graphs of:

(a) $y = \sin (x - \pi)$.

(b) $y = \cos (2x - \pi)$.

(c) $y = -3 \sin (2x - \frac{\pi}{2}) + 4$.

(d) $y = \cos (3x + \pi)$.

(e) $y = 3 \sin (\frac{1}{2}x - \pi) - 2$.

(f) $y = -2 \cos \frac{1}{2} x$.

I. G.

CONTENT: Circular Functions; Other Circular Functions

OBJECTIVE: The student will be able to:

(a) Define tangent, cotangent, secant, and cosecant in terms of the sine and cosine;

(b) Graph and state the domain and range of each.

ACTIVITIES:

(a) Use the values of cosine and sine to compute a table of values for tangent, cotangent, secant, cosecant of the special numbers between 0 and 2π.

(b) Graph two basic periods:

(1) $2 \tan \frac{x}{2}$.

(2) $y = -2 \cot 3x$.

(3) $y = \tan (\frac{1}{3}x + \frac{\pi}{6})$.

(4) $y = 2 \sec (2x - \frac{\pi}{6}) + 2$.

(5) $2 \csc \frac{1}{2}x$.

I. H.

CONTENT: Circular Functions; Identities

OBJECTIVE: The student will be able to state the basic identities and use them to verify other identities.

ACTIVITIES:

(a) State the eight basic identities.

(1) $\sin x \cdot \csc x = 1$

(2) $\cos x \cdot \sec x = 1$

(3) $\tan x \cdot \cot x = 1$

(4) $\tan x = \frac{\sin x}{\cos x}$

(5) $\cot x = \frac{\cos c}{\sin x}$

(6) $\sin^2 x + \cos^2 x = 1$

(7) $1 + \tan^2 x = \sec^2 x$

(8) $1 + \cot^2 x = \csc^2 x$

(b) Verify.

(1) $\cot x \cdot \sec x = \csc x$

(2) $\sin x (\sec x - \csc x) = \tan x - 1$

(3) $\frac{1 + \sin x}{\cos x} + \frac{\cos x}{1 + \sin x} = 2 \sec x$

(4) $\sec x - \cos x = \sin x \cdot \tan x$

II. A.

CONTENT: Angles and Their Measures; Definition of an Angle

OBJECTIVE: The student will be able to identify or define:

(a) An angle;

(b) Initial and terminal side of an angle;

(c) Co-terminal angles.

ACTIVITIES:

(a) Define an angle.

(b) Name the initial and terminal sides of the angle shown.

(c) Identify two co-terminal angles for the angle shown in part (b).

II. B.

CONTENT: Angles and Their Measures; Standard Position of an Angle

OBJECTIVE: The student will be able to sketch angles in standard position.

(a) an angle in standard position contains (P(1,1).

Find r (i.e., $\sqrt{x^2+y^2}$) and the degree measure of the angle.

(b) An angle in standard position contains P(-1,1). Find r and the degree measure of the angle.

(c) An angle in standard position contains P(0,-1). Find r and the degree measure of the angle.

III. A. CONTENT: Radian Measure; Conversions

OBJECTIVE: The student will be able to convert radian measure to degree measure and degree measure to radian measure.

ACTIVITIES:

(a) Express each of the following in radian measure.

(1) 60°

(2) 135°

(3) 270°

(4) 75°

(5) 300°

(b) Express each of the following radian measures as degree measures.

(1) $\frac{\pi}{2}$

(2) $\frac{5\pi}{4}$

(3) $\frac{11\pi}{6}$

(4) $\frac{7\pi}{8}$

(c) Find numerical values for each of the following without using a table.

(1) $\sin 45^\circ$

(2) $\sin 30^\circ \tan 45^\circ$

(3) $\cos 60^\circ \sec 45^\circ$

(4) $\cot 60^\circ \tan 30^\circ$

(5) $\sin 135^\circ$

III. B. CONTENT: Radian Measure; Length of an Arc of a Circle

OBJECTIVE: The student will be able to use radian measure to find the length of an arc of a circle.

ACTIVITIES:

(a) An arc of a circle has a corresponding central angle whose measure is 120 degrees. Find the length of the arc given that the radius of the circle is 12.

(b) The measure of a central angle of a circle is $\frac{\pi}{6}$ radians.

If the radius of the circle is 8", find the length of the arc.

III. C. CONTENT: Radian Measure; Area of a Sector

OBJECTIVE: The student will be able to use radian measure to find the area of a sector of a circle.

ACTIVITIES:

(a) The radian measure of an arc of a circle is .87. If the radius of the circle is 6 cm, find the area of the sector.

(b) The area of a sector of a circle is 20.3 sq. in. Find the radius of the circle given that the measure of the arc of the sector is $\frac{1}{2}$ radian.

III. D. CONTENT: Radian Measure; Velocity

OBJECTIVE: The student will be able to:

(a) Compute linear speed of a point moving on a circle;

(b) Find angular velocity of a point moving on a circle.

ACTIVITIES:

(a) The blade of a rotary lawn mower is 20 inches in diameter. If the blade turns 360 r.p.m., find, in ft./sec., the linear speed of the tip of the blade.

(b) The wheels of an automobile are 28 inches in diameter. If the automobile travels 65 m.p.h., find the angular velocity of the wheels in radians per second.

IV. A. CONTENT: Trigonometric Functions; Functions of Angles

OBJECTIVE: The student will be able to write the six trigonometric functions of angles.

ACTIVITIES:

(a) The point P(3,4) lies on the terminal side of the graphed angle D. Write the six trigonometric functions of D.

(b) The point Q(5,-12) lies on the terminal side of the graphed angle B. Write the six trigonometric functions of B.

IV. B. CONTENT: Trigonometric Functions; Functions of Special Acute Angles

OBJECTIVES: The student will be able to write the six trigonometric functions of angles whose measures are 30°, 45°, and 60°.

ACTIVITIES:

(a) Find numerical values for the following:

(1) $\sin 60^\circ + 2 \cos 30^\circ$

(2) $\sin 30^\circ \cos 60^\circ - \sin 45^\circ \cos 45^\circ$

(3) $2 \sin 60^\circ + \cos 30^\circ - \tan 45^\circ \cot 45^\circ$

(4) $\sec 45^\circ - \csc 30^\circ$

(b) Given $\{180^\circ \leq A < 270\}$ find:

(1) $\sec A$ given that $\cos A = \frac{-2}{3}$

(2) $\tan A$ given that $\csc A = \frac{-5}{3}$

(3) $\sin A$ given that $\cos A = \frac{-3}{8}$

IV. C. CONTENT: Trigonometric Functions; Cofunctions

OBJECTIVE: The student will be able to write functions of acute angles as functions of the complement of an angle.

ACTIVITIES:

(a) Write each of the following as a function of the complement of the angle:

(1) $\sin 10^\circ = \cos$ _____.

(2) $\tan 80^\circ =$ _____.

(3) $\cot 25^\circ =$ _____.

(4) $\cos 1^\circ +$ _____.

(5) $\sec 46^\circ =$ _____.

(b) Find angle A, given that:

(1) $\cos A = \sin (45^\circ - \frac{1}{2}A)$

(2) $\tan (45^\circ + A) = \cot A$

(3) $\cos 4A = \sin A$

IV. D. CONTENT: Trigonometric Functions; Reciprocal Relations

OBJECTIVE: The student will be able to write functions of angles as reciprocal functions of the same angle.

ACTIVITIES:

Use reciprocal relations to write each of the following as a function of the same angle.

(a) $\sin 10^\circ$

(b) $\tan 40^\circ$

(c) $\sec 15^\circ$

(d) $\csc 70^\circ$

(e) $\cos 25^\circ$

(f) $\cot 80^\circ$

IV. E. CONTENT: Trigonometric Functions; Quotient and Pythagorean Relations

OBJECTIVE: The student will be able to write trigonometric expressions as equivalent expressions by using the quotient and pythagorean relations.

ACTIVITIES:

Quotient Relations $\tan B = \frac{\sin B}{\cos B}$

$\cot B = \frac{\cos B}{\sin B}$

Pythagorean Relations $\sin^2 B + \cos^2 B = 1$

$\sec^2 B = 1 + \tan^2 B$

$\csc^2 B = 1 + \cot^2 B$

Use the fundamental identities of this section to answer the following. Assume that each angle is acute.

(a) If $\cos A = \frac{3}{5}$, find sin A. Find tan A.

(b) If $\tan B = \frac{12}{5}$, find cot B.

(c) If $\csc B = \frac{13}{5}$, find cot B.

(d) Express each of the six trigonometric functions as a function of sin A.

IV. F. CONTENT: Trigonometric Functions; Tables and Linear Interpolation

OBJECTIVE: The student will be able to find functions of angles by using a table and by linear interpolation.

ACTIVITIES:

(a) Use a table to find:

(1) sin 20° 10

(2) tan 84° 40'

(3) cot 48° 20'

(4) cot 14° 50'

(b) Use interpolation to find:

(1) sin 10° 13'

(2) cot 48° 37'

(3) tan 27° 27'

(4) cot 78° 54'

(c) Find acute angle A, given that:

(1) sin A = .0873

(2) cot a = 1.7560

(3) cot A = .5534

(4) tan A = 1.1650

V. A.

CONTENT: Trigonometric Functions of Any Angle; Reference Angle

OBJECTIVE: The student will be able to express the function of any angle as a function of a reference angle.

ACTIVITIES:

(a) Express each of the following as the same function of a first quadrant angle.

(1) $\cot 135^\circ =$

(2) $\tan 220^\circ =$

(3) $\csc \frac{7\pi}{9}$

(4) $\cot 320^\circ =$

(5) $\sin \frac{23\pi}{18}$

(6) $\tan 335^\circ =$

(7) $\sin (-100^\circ) =$

(8) $\tan (-120^\circ) =$

(9) $\sin (-330^\circ) =$

(b) Express each of the following as a function of an angle between 0 and 45°:

(1) $\sin 100^\circ$

(2) $\cot 310^\circ$

(3) $\tan 120^\circ$

(4) $\sec 250^\circ$

(5) $\cot 95^\circ$

(6) $\tan (-100^\circ)$

(7) $\sec (-130^\circ)$

(8) $\tan (-420^\circ)$

(9) $\csc 260^\circ$

(10) $\sin (-225^\circ)$

(c) Find (in simplest form) numerical values for each of the following. Do not use a table.

(1) $\sin 150^\circ \cos 300^\circ$

(2) $\sin 510^\circ \tan 225^\circ$

(3) $\cos (-60^\circ) \csc 315^\circ$

(4) $\sin (-45^\circ) \sin 30^\circ$

(5) $\tan 120^\circ + \cos 120^\circ$

(d) If $0 \leq A < 360^\circ$, find all angles for which:

(1) $\sin a = \frac{1}{2}$

(2) $\tan A = \sqrt{3}$

(3) $\sec A = \frac{2}{\sqrt{3}}$

(4) $\cos A = \frac{\sqrt{3}}{2}$

(5) $\tan A = \sqrt{3}$

(6) $\cos A = -\sqrt{3}$

(7) $\csc A = -\sqrt{2}$

(8) $\tan A = -1$

(9) $\sec A = -\sqrt{2}$

(e)

(1) Find A given that $\sin A = \frac{1}{2}$ and $90^\circ < A < 180^\circ$

(2) Find A given that $\tan A = 3$ and $0^\circ < A < 90^\circ$

(3) Find A given that $\cot A = -3$ and $90^\circ < A < 180^\circ$

(4) Find $\sin A$ given that $\sec A = \frac{5}{4}$ and $270^\circ < A < 360^\circ$

(5) Find $\cos A$ given that $\tan A = \frac{-5}{6}$ and $270^\circ < A < 360^\circ$

(6) Find $\tan A$ given that $\csc A = \frac{-7}{4}$ and $180^\circ < A < 270^\circ$

V. B. CONTENT: Trigonometric Functions of Any Angle; Quadrantal Angles

OBJECTIVE: The student will be able to find the function of angles whose measures are multiples of 0°, 90°, 180°, 270°.

ACTIVITIES: Find numerical values for each of the following:

(a) $\sin 180^\circ$

(b) $\tan 180^\circ \sin 270^\circ + \tan 45^\circ \cos 180^\circ$

(c) $\sec 180^\circ \csc 270^\circ - \cos 270^\circ \sin 90^\circ$

(d) $\cos 540^\circ + \sin 720^\circ$

(e) $\sec 360^\circ \tan (-135^\circ) + \cos 360^\circ \cot (-225^\circ)$

(f) $\sin 180^\circ \cos 45^\circ - \tan 135^\circ \sin 210^\circ + \sin {}^2 90^\circ$

(g) $\sec 60^\circ \cos 90^\circ - \dfrac{\tan 45^\circ}{\cot 225^\circ}$

(h) $\csc (-135^\circ) \sec 180^\circ + \sin 180^\circ \cos 225^\circ$

V. C. CONTENT: Trigonometric Functions of Any Angle; Formulas; Sum and Difference Formulas

OBJECTIVE: The student will be able to apply the formulas for the functions of the sum or difference of two angles.

ACTIVITIES: Formulas:

$$\sin (a + B) = \sin a \cos B + \cos a \sin B$$

$$\sin (a - B) = \sin a \cos B - \cos a \sin B$$

$$\cos (a + B) = \cos a \cos B - \sin a \sin B$$

$$\cos (a - B) = \cos a \cos B + \sin a \sin B$$

$$\tan (a + B) = \frac{\tan a + \tan B}{1 - \tan a \tan B}$$

$$\tan (a - B) = \frac{\tan a - \tan B}{1 + \tan a \tan B}$$

(a) Find $\sin 105^\circ$ by using $\sin (60^\circ + 45^\circ)$.

(b) Find $\cos 15^\circ$ by using $\cos (45^\circ - 30^\circ)$.

(c) Find $\sin 15^\circ$ by using functions of 45° and 30°.

(d) Find $\sin 75^\circ$ without using a table.

(e) If $\cos A = \frac{4}{5}$, $270^\circ < A < 360^\circ$, and $\sin B = \frac{-12}{13}$, $180^\circ < B < 270^\circ$; find $\cos (A + B)$, $\cos (A - B)$, $\sin (A + B)$, $\sin (A - B)$

(f) If $\cot A = \frac{1}{2}$, $180^\circ < A < 270^\circ$ and $\sin B = \frac{2}{\sqrt{13}}$, $90 < B < 180^\circ$; find $\tan (A + B)$ and $\tan (A - B)$.

V. C. (2) CONTENT: Trigonometric Functions of Any Angle; Formulas; Half-Angle and Double-Angle Formulas

OBJECTIVE: The student will be able to apply half-angle and double-angle formulas.

ACTIVITIES:

Formulas:

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\sin \left(\frac{A}{2}\right) = \pm \sqrt{\frac{1 - \cos A}{2}}$$

$$\cos \left(\frac{A}{B}\right) = \pm \sqrt{\frac{1 + \cos A}{2}}$$

$$\tan \frac{A}{2} = \pm \sqrt{\frac{1 - \cos A}{1 + \cos A}} = \frac{1 - \cos A}{\sin A} = \frac{\sin A}{1 + \cos A}$$

(a) Find $\sin 60^\circ$ by using a double-angle formula.

(b) If $\sin A = \frac{4}{5}$, $0^\circ < A < 90^\circ$, find $\sin 2A$, $\cos 2A$ and $\tan 2A$.

(c) Find $\cos 45^\circ$ from functions of 90°.

(d) If $\tan B = \frac{12}{13}$, $0^\circ < B < 90^\circ$, find $\tan 2B$.

Find $\tan(\frac{B}{2})$.

(e) If $\cos A = \frac{4}{5}$, $270^\circ < A < 360^\circ$, find $\cos(\frac{A}{2})$.

Find $\cos 2A$.

(f) If $\sin A = \frac{-5}{13}$, $270^\circ < A < 360^\circ$, find $\sin 2A$.

Find $\sin(\frac{A}{2})$.

V. D. CONTENT: Trigonometric Functions of Any Angle; Identities

OBJECTIVE: The student will be able to verify that expressions are identities by using trigonometric formulas.

ACTIVITIES: Verify each of the following identities:

(a) $\tan^2\theta \cos^2\theta + \cot^2\theta \sin^2\theta = 1$

(b) $\frac{\sin\theta}{1+\cos\theta} + \frac{\cos\theta}{\sin\theta} = \csc\theta$

(c) $\frac{\sec^2\theta - 6\tan\theta + 7}{\sec^2\theta - 5} = \frac{\tan\theta - 4}{\tan\theta + 2}$

(d) $\sec^4\theta - \sec^2\theta = \frac{1}{\cot^4\theta} + \frac{1}{\cot^2\theta}$

(e) $\frac{1}{2}\sin 2\theta = \frac{\tan\theta}{1+\tan^2\theta}$

(f) $\frac{\sin\theta + \sin 3\theta}{\cos\theta + \cos 3\theta} = \tan 2\theta$

(g) $1 + \cos 2\theta = \frac{2}{1+\tan^2\theta}$

(h) $\cot\theta = \frac{\sin 2\theta}{1 - \cos 2\theta}$

(i) $\csc\theta \sec\theta = 2\csc 2\theta$

VI. A

CONTENT: Inverse Trigonometric Relations; General Values

OBJECTIVE: The student will be able to find general values of inverse relations.

ACTIVITIES:

(a) Find all angles θ for which:

(1) $\theta = \arcsin \frac{1}{2}$

(2) $\theta = \arccos \frac{\sqrt{2}}{2}$

(3) $\theta = \operatorname{arcsec} 2$

(4) $\theta = \arccos \left(-\frac{\sqrt{3}}{2}\right)$

(5) $\theta = \arctan (-1)$

(6) $\theta = \arctan .4142$

(b) Assume that all angles are <u>acute</u> and find:

(1) $\sin \left(\arcsin \frac{\sqrt{3}}{2}\right)$

(2) $\tan \left(\arctan \frac{2}{3}\right)$

(3) $\tan \left(\arcsin \frac{4}{5}\right)$

(4) $\sin (\arctan 1) + \cos \left(\arctan \frac{\sqrt{3}}{3}\right)$

VI. B.

CONTENT: Inverse Trigonometric Relations; Graphs of Inverse Trigonometric Functions

OBJECTIVE: The student will be able to graph inverse trigonometric functions.

ACTIVITIES:

(a) Sketch the graph of y = Arcsin x. Identify the domain and range of the functions.

(b) Sketch the graph of y = Arccos x. Identify the domain and range of the function.

(c) Sketch the graph of y = Arctan x. Identify the domain and range of the function.

VI. C.

CONTENT: Inverse Trigonometric Relations; Principal Values of Inverse Functions

OBJECTIVE: The student will be able to find principal values of inverse trigonometric functions.

ACTIVITIES: Find the principal value of each of the following:

(a) Arcsin $\frac{1}{2}$

(b) Arccos $\frac{(-\sqrt{2})}{2}$

(c) Arctan (-1)

(d) Arcsin (-.5125)

(e) $\text{Cos}^{-1}\ (\frac{\sqrt{3}}{2})$

(f) $\text{Tan}^{-1}\ 4$

VII. A. CONTENT: Trigonometric Equations; Principal Values

OBJECTIVE: The student will be able to find principal values of trigonometric equations.

ACTIVITIES:

Find the principal values of each of the following:

(a) $2 \sin x - 1 = 0$

(b) $\sqrt{3} \tan x = -1$

(c) $\sin^2 x - 3 \sin x + 2 = 0$

(d) $2 \sin x \cos x - \cos x = 0$

(e) $\sec x \tan x - \tan x + 2 \sec x - 2 = 0$

VII. B. CONTENT: Trigonometric Equations ; General Values

OBJECTIVE: The student will be able to find general values of trigonometric equations.

ACTIVITIES:

Find the general values of the following equations:

(a) $y = \text{arcsec } 2$

(b) $3 \tan x - \cos x = 0$

(c) $\sin x + \cos x \sin x = 0$

(d) $2 \sin^2 x - \sin x + 1 = 0$

(e) $\sqrt{3} \sec x \sin x - 2 \sec x + \sqrt{3} \sin x = 2$

(f) $\cos x - \sin x = 1$

(g) $2 \sin^2 x - \cos x - 1 = 0$

(h) $\sin^2 x - 2 \sin x - 1 = 0$

VII. C. CONTENT: Trigonometric Equations: Restricted Solutions

OBJECTIVE: The student will be able to find restricted solutions of trigonometric equations.

ACTIVITIES: Solve for x, $0 \leq x < 360^\circ$

(a) $\sin (x - 30) = \frac{1}{2}$

(b) $5 \sec^2 x + 2 \tan x - 8 = 0$

(c) $2 \cos 2x - \sin x + 1 = 0$

(d) $2 \sin x \tan x + 2 \sin x - \tan x - 1 = 0$

(e) $\tan^3 x - 3 \tan x = 0$

(f) $\sin x - 2 \cos x = 1$

(g) $\sin 2x \cos x + \cos 2x \sin x = \frac{1}{2}$

(h) $\sin^2 x - \cos^2 x = 0$

VIII. A. CONTENT: Applications of Trigonometry; Right Triangles

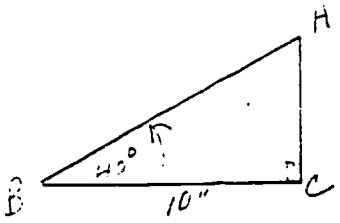
OBJECTIVE: The student will be able to solve right triangles given

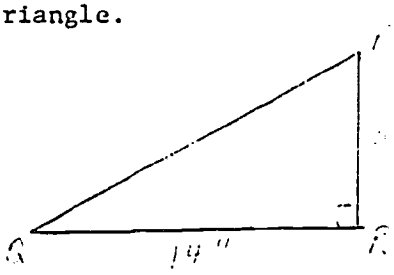
(a) Two sides;

(b) An angle and a side.

ACTIVITIES:

(a) Solve each right triangle.





(b) The measure of each base angle of an isosceles triangle is 72 degrees. If the length of the base is 10 cm, find the length of the altitude to the base and the length of the congruent sides of the triangle.

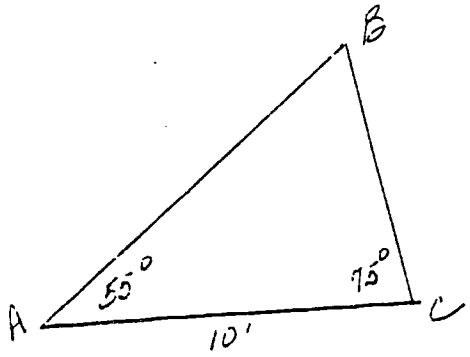
(c) A monument 200 feet high casts a shadow 325 feet long. Find the angle of elevation of the sun.

VIII. B. CONTENT: Applications of Trigonometry; Law of Sines and Law of Cosines

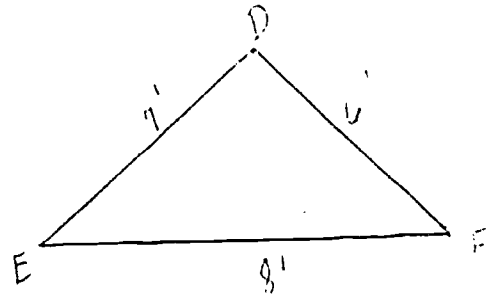
OBJECTIVE: The student will be able to solve triangles by applying the Law of Sines and the Law of Cosines.

ACTIVITIES:

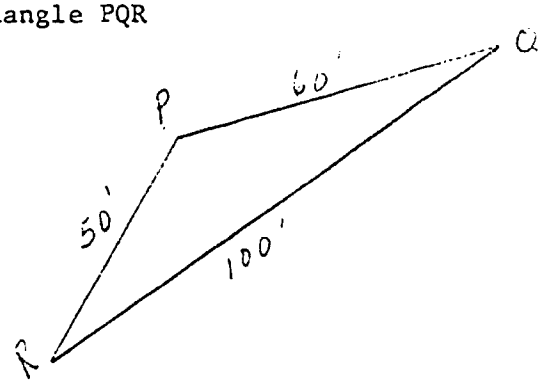
(a) Solve triangle ABC.



(b) Solve triangle DEF.



(c) Solve triangle PQR



VIII. C. CONTENT: Application of Trigonometry ; Use of Calculators

OBJECTIVE: The student will be able to perform calculations involving trigonometric functions using the calculator.

ACTIVITIES:

(a) Given triangle ABC:

(1) a = 22.34

(2) b = 13.74

(3) c = 10.15

Find angle B.

(b) Given triangle ABC:

(1) A = 70° 13'

(2) B = 52° 48'

(3) a = 640

Find side b.

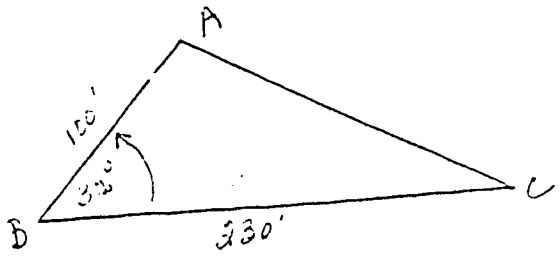
VIII. D.

CONTENT: Applications of Trigonometry; Area of Triangles

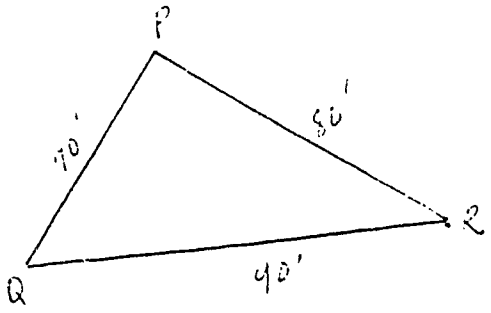
OBJECTIVE: The student will be able to find the area of a triangle given the lengths of three sides or of two sides and the measure of the included angle of the triangle.

ACTIVITIES:

(a) Find the area of triangle ABC.



(b) Find the area of triangle PQR.



IX. A. CONTENT: Polar Coordinates; Conversion

OBJECTIVE: The student will convert from rectangular to polar coordinates and vice-versa.

ACTIVITIES:

(a) Convert to polar coordinates

(1) $(1,1)$

(2) $(3,-4)$

(b) Convert to rectangular coordinates.

(1) $(2,\frac{5\pi}{6})$

(2) $(-3,210^{o})$

IX. B. CONTENT: Polar Coordinates; Graphing

OBJECTIVE: The student will sketch graphs of relations in polar form

ACTIVITIES:

(a) $r = 2 \cos \theta$

(b) $r = 2(1 + \cos \theta)$

(c) $r = 2 \sin 2 \theta$

X. A. CONTENT: Complex Numbers; Definition and Properties

OBJECTIVE: The student will be able to:

(a) Define complex, numbers;

(b) Perform fundamental operations on complex numbers.

ACTIVITIES:

(a) Write each of the following in the form of $x + yi$

(1) $(i)^6$

(2) $(1 + i)(2 - i)$

(3) $(2 + 3i) - (4 - 6i)$

(4) $\frac{1 - i}{5 - 2i}$

(5) $(2 - 3i)^2$

(6) $|3 - i|$

(b) Solve for x and y

(1) $3x + 15i = 6 - yi$

(2) $x - yi = 2 + 3i$

(3) $x + y - 4 = - xi + 2yi + i$

(4) $x + yi = (3 + i)(1 - 3i)$

(5) $2x + y + (3x - y)\, i = 4 + i$

X. B. CONTENT: Complex Numbers; Rectangular and Polar Forms

OBJECTIVE: The student will be able to write complex numbers in rectangular and in polar form.

ACTIVITIES:

(a) Represent each complex number Z as a point in the plane and find the amplitude and modulus. Express the complex number in polar form.

(1) $Z = 1 + i$

(2) $Z = \sqrt{3} - 3i$

(3) $Z = 3i$

(4) $Z = 5$

(5) $Z = -1 + i$

(6) $Z = \frac{1}{2} + i\frac{\sqrt{3}}{2}$

(7) $Z = 1$

X. C. CONTENT: Complex Numbers; Multiplication and Division in Polar Form

OBJECTIVE: The student will be able to multiply and divide complex numbers in polar form.

ACTIVITIES:

(a) Perform the indicated operations and express each answer in rectangular form.

(1) $2(\cos 30^\circ - i \sin 30^\circ) \cdot 3(\cos 30^\circ + i \sin 30^\circ)$

(2) $\frac{8(\cos 135^\circ + i \sin 135^\circ)}{2(\cos 45^\circ + i \sin 45^\circ)}$

(3) $(5 \text{ cis } 80^\circ) \cdot (4 \text{ cis } 40^\circ)$

(4) $(6 \text{ cis } 85^\circ) \div (3 \text{ cis } 25^\circ)$

(5) $(10 \text{ cis } 90^\circ) \div (2 \text{ cis } 45^\circ)$

(b) Express each complex number in polar form and then perform the indicated operations.

(1) $(1 + i)(1 - i\sqrt{3})$

(2) $(2 - 2i)(1 + \sqrt{3}i)$

(3) $\frac{\sqrt{3} + 3i}{2 - 2i}$

(4) $\frac{(1 - i)(-1 + i\sqrt{3})}{(\sqrt{3} + i)}$

(5) $(1 + i)(-2 + 2i)$

(6) $\frac{1 + i}{1 - i}$

(7) $\frac{-1 + i}{1 + \sqrt{3}i}$

X. D. CONTENT: Complex Numbers; DeMoivre's Theorem

OBJECTIVE: The student will be able to use DeMoivre's Theorem to raise complex numbers to integral powers.

ACTIVITIES: Use DeMoivre's Theorem and express the following complex numbers in rectangular form:

(a) $(2 \text{ cis } 20^\circ)^3$

(b) $(3 \text{ cis } 15^\circ)^2$

(c) $(\text{cis } 18^\circ)^5$

(d) $(\sqrt{3} + i)^4$

(e) $(-i)^7$

(f) $(1 + i)^4$

(g) $\frac{(-1 + i\sqrt{3})^7}{2}$

(h) $(-\sqrt{3} + i)^6$

X. E. CONTENT: Complex Numbers; Roots of Complex Numbers

OBJECTIVE: The student will be able to find all roots of complex numbers.

ACTIVITIES:

(a) Find the 3 cubic roots of 8i. Express each in rectangular form.

(b) Find the 4 fourth roots of 16. Express each in rectangular form.

(c) Find the 4 fourth roots of $-8 - 8\sqrt{3}i$. Express each in rectangular form.

(d) Find the 3 cubic roots of $-1 + i$.

(e) Find the fifth roots of -1.

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